Sets

SET07106 Mathematics for Software Engineering

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Outline

Examples of sets

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\{\triangle, \square, \bigcirc, \triangle, \square, \bigcirc, \triangle, \square, \bigcirc\}$

$\{\heartsuit, \spadesuit, \diamondsuit, \clubsuit\}$

$\{1, 2, 3, 4, ...\}$

$\{2, 3, 5, 7, 11, ...\}$

Set operators: elements

Sets can be defined *extensionally*, i.e. by listing their elements.

$\triangle \in \{\triangle, \square, \bigcirc\}$

$\triangle \notin \{\triangle, \square, \bigcirc\}$

Set operators: equality

Two sets are equal if they contain the same elements.

$\{\triangle, \square, \bigcirc\} = \{\bigcirc, \square, \triangle\}$

$\{\triangle, \square, \bigcirc\} \neq \{\bigcirc, \square, \triangle\}$

Defining a set: $S := \{\square, \triangle, \bigcirc\}$

The empty set: $\emptyset := \{\}$

Set operators: union, intersection and difference

$A \cup B$ contains elements that are in A or B or in both.
$A \cap B$ contains elements that are in A and B.
$A \setminus B$ contains elements that are in A but not in B.

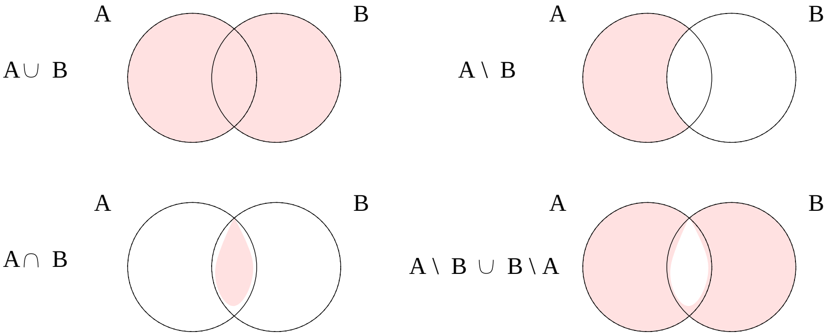
$\{\color{blue}\square, \triangle, \bigcirc\color{black}\} \cup \{\color{red}\square, \triangle, \bigcirc\color{black}\} = \{\color{blue}\square, \triangle, \bigcirc\color{black}, \color{red}\square, \triangle, \bigcirc\color{black}\}$

$\{\color{blue}\square, \triangle, \bigcirc\color{black}, \color{red}\square\color{black}\} \cap \{\color{red}\square, \triangle, \bigcirc\color{black}\} = \{\color{red}\square\color{black}\}$

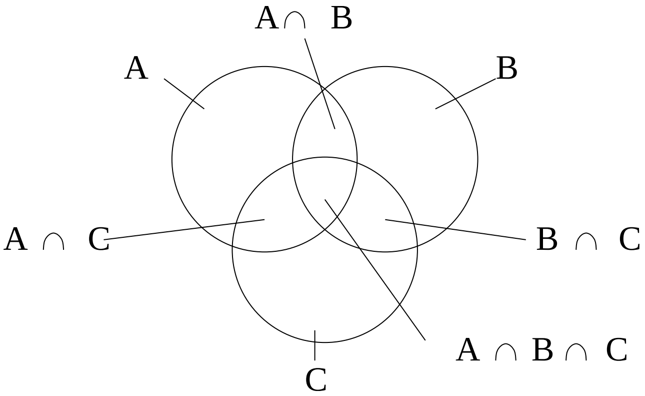
$\{\color{blue}\square, \triangle, \bigcirc\color{black}, \color{red}\square\color{black}\} \setminus \{\color{red}\square, \triangle, \bigcirc\color{black}\} = \{\color{blue}\square, \triangle, \bigcirc\color{black}\}$

$\{\color{blue}\square, \triangle, \bigcirc\color{black}\} \cap \{\color{red}\square, \triangle, \bigcirc\color{black}\} = \emptyset$ (i.e. the sets are *disjoint*)

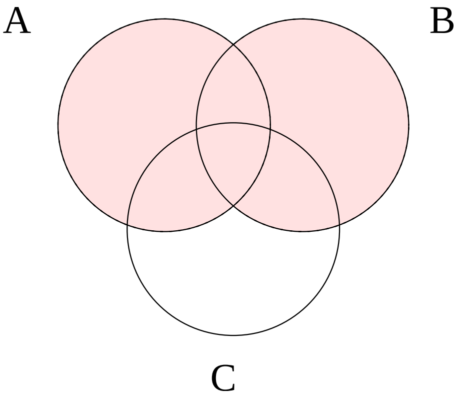
Venn diagrams



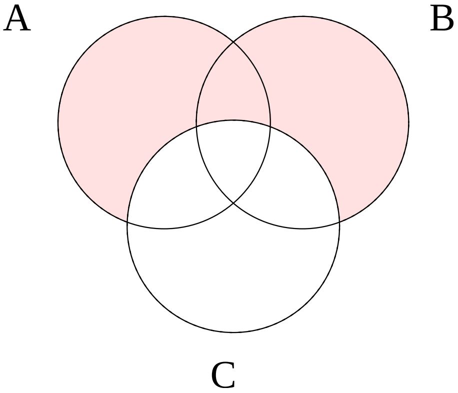
Venn diagram for 3 sets



Venn diagrams: $A \cup B$



Venn diagrams: $(A \cup B) \setminus C$



Set operators: subsets

$\{\triangle, \square, \bigcirc, \triangle\} \subset \{\triangle, \square, \bigcirc, \triangle, \square, \bigcirc, \triangle, \square, \bigcirc\}$

$\subseteq$ means "either equal or subset"

Laws:

$A \subseteq B \iff A \subset B$ or $A = B$

$A \subseteq B \iff A \cap B = A$

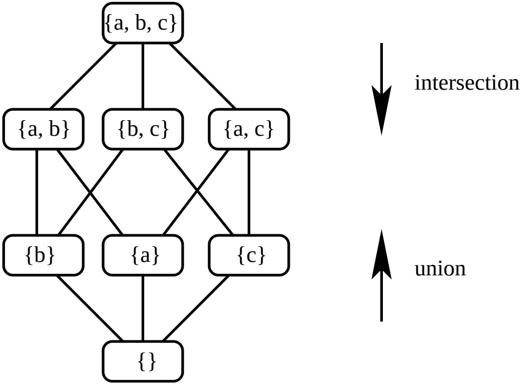
$A = B \iff A \subseteq B$ and $B \subseteq A$

Exercise

Write down all subsets of $\{\triangle, \square, \bigcirc\}$.

(This is called the **power set** of $\{\triangle, \square, \bigcirc\}$.)

A lattice of all subsets of $\{a, b, c\}$



Exercise: which of these are true?

$\{\triangle, \square, \bigcirc, \triangle, 1\}$ is a set.

$\{\square, \triangle, \square, \bigcirc\}$ is a set.

$\{\triangle\} \in \{\bigcirc, \square, \triangle\}$

$1 \notin \{\bigcirc, \square, \triangle\}$

$\{\square, \triangle, \bigcirc\} \subseteq \{\square, \triangle, \bigcirc\}$

$\{\square, \triangle, \bigcirc\} \subset \{\square, \triangle, \bigcirc\}$

Associative

Brackets can be moved or left off completely.

$(A \cap B) \cap C = A \cap (B \cap C)$
$(A \cup B) \cup C = A \cup (B \cup C)$

This is also true for integer addition $(2 + 3 + 5)$, but not if different operators are used: $2 \times (3 + 5) \neq (2 \times 3) + 5$.

Commutative

The order can be changed.

$A \cap B = B \cap A$
$A \cup B = B \cup A$

This is also true for integer addition $(2 + 5)$, but not for subtraction: $2 - 5 \neq 5 - 2$.

Distributive

This explains how different operators can be combined.

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

This is also true for integer addition/multiplication:
$2 \times (3 + 4) = 2 \times 3 + 2 \times 4$.
But: $2 + (3 \times 4) \neq (2 + 3) \times (2 + 4)$.

Idempotent

Repeated operation has no effect.

$A \cap A = A$
$A \cup A = A$

In general this is not true for integers.
For which n is $n + n = n$? For which m is $m \times m = m$?

Transitive

The operation continues across an ordering.

$A \subseteq B$ and $B \subseteq C$ implies $A \subseteq C$
$A \supseteq B$ and $B \supseteq C$ implies $A \supseteq C$

The same is true for $\leq$ and $\geq$ among integers.
$2 \leq 5$ and $5 \leq 7$ implies $2 \leq 7$.

Exercise

For each of the operators determine which properties they have:

	associative	commutative	distributive	idempotent	transitive
$\cap$					
$\cup$					
$=$					
$\in$					
$\subseteq$					
$\setminus$					
$\subset$					

Modelling mathematics with a programming language

⋆ In mathematics:
Entities (elements, sets, etc) are abstract.
They exist outside of space and time.

⋆ In programming languages:
All entities are real.
They exist in a space (in memory, on a drive, etc) in real time.

Programming languages can only approximate mathematical entities. It is not possible to conduct pure mathematics with a computer.

Modelling sets with a programming language

⋆ In mathematics:
The elements in a set can occur in any order. No element can occur more than once. Sets can be infinite.

⋆ In programming languages:
A natural data type is lists, arrays, bags.

```
list = [1, 2, 3]
```

Lists are finite, have a fixed order and can contain the same value twice.

Sets in Python

```
students = Set(['Joe', 'Jane', 'Mary', 'Pete'])
```

	mathematics	Python
Union	$\cup$	|
Intersection	$\cap$	&
Difference	$\setminus$	-
Element	$\in$	in