Classification of Meronymy by Methods of Relational Concept Analysis

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Abstract

This paper introduces a method for classifying the meronymy relation based on quantificational tags. It is an example for an application of Relational Concept Analysis, which is an extension of Formal Concept Analysis, in the field of computational linguistics. Therefore this paper does not report the complete results of an investigation, but it tries to give ideas for further research using these methods.

1 Introduction

Different authors have tried to classify the meronymy relation. Winston et al. (1987) distinguish six kinds of the meronymy relation which are separated by so-called 'relation elements'. Chaffin & Herrmann (1988) distinguish eight different kinds by using relation elements, but neither their kinds nor their relation elements coincide with the classification of Winston et al. A classification by Iris et al. (1988) which is based on four elementary models also only partly coincides with the other classifications. Cruse (1986) uses a different approach, he identifies four kinds of the meronymy relation based on quantificational differences. This approach is further developed by Woods (1991) who suggests that a semantic relation consists of a *quantificational tag* and a *relational component*. In this paper Woods' idea is embedded into the formal analysis of conceptual hierarchies. A theory for conceptual data structuring, called Formal Concept Analysis, has been developed for more than sixteen years at the Technische Hochschule Darmstadt (Ganter & Wille, 1996). It defines a concept based on its *extent*, which denotes the set of the *formal objects* of the concept, and on its *intent*, which denotes the set of the *formal attributes* of the concept. Concepts can either be represented in *formal contexts* which are cross-tables of the relation between objects and attributes or in the form of mathematical lattice diagrams. While Woods defines a relation r among 'instances' which leads to a relation R among 'classes', in the framework of Formal Concept Analysis, the relation r is defined among formal objects and inherited by concepts as relation R. This leads to Relational Concept Analysis, which is therefore the extension of Formal Concept Analysis to a more general theory that includes additional relations.

Formal and Relational Concept Analysis have already been used for applications in various subject areas. They are applicable to linguistics in several ways. First, they can facilitate the formalization of linguistic items by restricting lexical data to fixed contexts and specifying the role of each item in the context. While constructing formal contexts, the linguist has to determine if the formal objects are denotata of word forms, word forms or disambiguated word forms (Priss, in prep.2). The formal attributes can be attributes of denotata or connotational attributes. Depending on the selection of objects and attributes, the resulting formal concepts can represent denotative concepts, meta-concepts, word concepts or others.

Second, Woods' idea of inheritance of semantic relations (for example, from subconcepts to superconcepts) can be formalized, further investigated, and formally proved in the framework of Relational Concept Analysis (Priss, in prep.1). This can be applied to any semantic network that has hierarchical relations. Third, irregularities in the implementation of the semantic relations of a lexical database can be found and corrected. This is shown for the lexical database, WordNet, by Priss (in prep.3). Fourth, in this paper formal properties of semantic relations are used to obtain classificational attributes. This is demonstrated using the quantificational tags of the meronymy relation.

2 Formal Concept Analysis

Formal Concept Analysis, as a theory of data structuring, starts with the notion of a *formal context* that is defined as a triple (G, M, I) where G is the set of *formal objects* (Gegenstände), M is the set of *formal attributes* (Merkmale), and I is a binary relation between G and M for which gIm is interpreted as 'the object g has the attribute m' (Ganter & Wille, 1996). The *prime-operator* yields all common attributes of a set of objects $X \subseteq G$, $X' := \{m \in M \mid gIm \text{ for all } g \in X\}$ or all common objects of a set of attributes $Y \subseteq M$, $Y' := \{g \in G \mid gIm \text{ for all } m \in Y\}$. A pair (X, Y) is said to be a *formal concept* of the context (G, M, I) if $X \subseteq G, Y \subseteq M, X = Y'$, and $Y = X'$. If (X, Y) is a concept, then X is called the *extent*, Y is called the *intent* of the concept. A main characteristic of a concept is the fact that the extensional and the intensional definition are equivalent. In Figure 1 the classification of meronymy by Chaffin & Herrmann is used as an example. The ‘relation families’ are interpreted as the formal objects, the relation elements as the formal attributes of the formal context.

	inclusion	partive	attached	component	property	possession	homogen.	social	locative
functional object	X	X	X	X	X	X			
collection	X	X			X	X	X		
group	X	X			X	X	X	X	
ingredient	X	X		X	X	X			X
functional location	X	X	X	X	X	X			
organization	X	X	X	X	X	X		X	
measure	X	X					X		
place	X	X				X			X

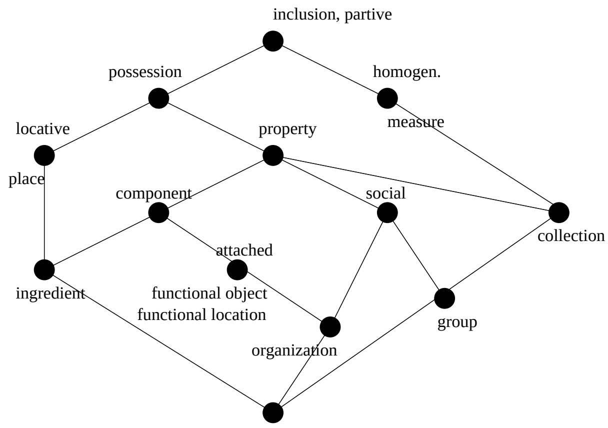

Figure 1: Formal context and line diagram of its concept lattice

The set of all concepts of (G, M, I) is denoted by $\mathcal{B}(G, M, I)$. The most important structure on $\mathcal{B}(G, M, I)$ is given by the subconcept-superconcept relation that is defined as follows: the concept (X_1, Y_1) is a subconcept of the concept (X_2, Y_2) if $X_1 \subseteq X_2$, which is equivalent to $Y_2 \subseteq Y_1$; (X_2, Y_2) is then a superconcept of (X_1, Y_1). This definition yields an order relation ‘$\leq$’ on $\mathcal{B}(G, M, I)$ with which the set of all concepts forms a lattice $\underline{\mathcal{B}}(G, M, I)$. Lattices are effectively visualized by line diagrams. Each object g labels the concept γg in the line diagram that is the smallest concept the object belongs to. Dually, an attribute m labels μm the largest concept it belongs to. The advantage of the lattice representation is that the similarity of the ‘relation families’ does not have to be calculated using statistical methods as in Chaffin & Herrmann’s paper. But the similarities of the relation families to each other can be examined by investigating the lattice diagram. It becomes, for example, obvious that ‘functional object’ and ‘functional location’ are not properly discriminated. Maybe ‘functional location’ should also have the relation element ‘locative’. It can be observed that ‘group’ is a subconcept of ‘collection’, because a ‘group’ has all the relation elements of a ‘collection’ but furthermore it has the relation element ‘social’. These examples show that the lattice representation can serve as the basis for a scientific discussion on subjects whose structure would not be transparent otherwise.

3 Relational Concept Analysis

Relational Concept Analysis considers additional relations among objects or attributes besides the conceptual hierarchy. It can be interpreted as an extension of Woods’ quantificational tags, relational components, and inheritances. In what follows, only binary relations $r \subseteq G \times G$ are considered. These relations are transferred to relations among concepts, i. e., $R \subseteq \mathcal{B}(G, M, I) \times \mathcal{B}(G, M, I)$, according to the following definitions. The quantifiers that are used in the definitions can be natural language or mathematical expressions, such as $||$all$||$, $||$at least 1$|| =: || \geq 1||$, or $||$exactly 1$|| =: ||1||$ (for more details on natural language quantifiers see Westerstahl (1989)). For the context (G, M, I), the relation $r \subseteq G \times G$, the extents $\underline{B}_1$, $\underline{B}_2$ of the concepts B_1, B_2, and the quantifiers Q^i, $1 \leq i \leq 4$, are defined

$$B_1 \ R^r[Q^1, Q^2;] \ B_2 :\Longleftrightarrow Q^1_{g_1 \in \underline{B}_1} Q^2_{g_2 \in \underline{B}_2} : g_1 r g_2 \tag{1}$$

$$B_1 \ R^r[; Q^3, Q^4] \ B_2 :\Longleftrightarrow Q^3_{g_2 \in \underline{B}_2} Q^4_{g_1 \in \underline{B}_1} : g_1 r g_2 \tag{2}$$

$$B_1 \ R^r[Q^1, Q^2; Q^3, Q^4] \ B_2 :\Longleftrightarrow \tag{3}$$
$$B_1 \ R^r[Q^1, Q^2;] \ B_2 \text{ and } B_1 \ R^r[; Q^3, Q^4] \ B_2$$

Each relation r leads therefore to several different relations R^r among concepts. Relations among concepts based on a relation $r \subseteq M \times M$ among attributes are defined similarly. In linguistic applications they can be used, for example, to describe antonymy, but this definition will not be considered in this paper. The formalization can be best understood through an example: ‘all door-handles are parts of doors’ states a meronymy relation between door-handles and doors. More precisely it means that all objects that belong to the concept ‘door-handle’ have an object in the concept ‘door’ so that the meronymy relation holds between them. The variables in equivalence (1) are therefore $Q^1 := ||$all$||$, $Q^2 := || \geq 1||$, B_1

is the concept ‘door-handle’, B_2 is the concept ‘door’, and r is the relation ‘is part of’. Equivalence (2) could be ‘there is at least one door which has a handle’, because ‘all doors have to have handles’ is probably not true.

The case where $Q^1 = Q^2 = ||\geq 1||$ and therefore also $Q^3 = Q^4 = ||\geq 1||$ is abbreviated as R_0^r. It is the minimal relation where at least one pair of objects is in relation r to each other, because if Q^1 or Q^2 equals $||\geq 0||$, then it is not known whether there is a single pair of objects with relation r at all. In most applications Q^1 and Q^3 equal $||\text{all}||$, therefore ‘$R^r[||\text{all}||, Q^2; ||\text{all}||, Q^4]$’ is abbreviated as $R^r_{(Q^4;Q^2)}$. Furthermore, the vertical lines ‘$||$’ can be left out for Q^4 and Q^2 in the subscript of $R^r_{(Q^4;Q^2)}$. Thus $B_1 R^r_{(Q^4;Q^2)} B_2$ is read as: for all elements g_1 in the extent of B_1 exist Q^2 elements g_2 in the extent of B_2 with $g_1 r g_2$ and for all elements g_3 in the extent of B_2 exist Q^4 elements g_4 in the extent of B_1 with $g_3 r g_4$. With Woods’ terminology, the subscript $(Q^4; Q^2)$ of R becomes the quantificational tag, the superscript r the relational component. Besides its applications to the modeling of lexical databases, this formalization can be used to describe functions $R^r_{(\geq 0;1)}$, bijections $R^r_{(1;1)}$, or Cartesian products $R^r_{(\text{all};\text{all})}$.

For linguistic applications the $||\text{some}_1||$- and the $||\text{several}||$-quantifiers are useful. $||\text{some}_1||$ is used for mass nouns, such as ‘a sausage contains some meat’. The index ‘1’ refers to the singular meaning of ‘some’ (there is some meat) in contrast to the plural meaning (there are some people). (It should be noted that all these examples have to be understood in a prototypical sense: ‘a prototypical sausage contains meat’, and so on.) $||\leq \text{some}_1||$ means there are none or some, but never all (‘a pizza contains some meat or no meat at all’); $||\text{some}_1||$ means there are exactly some, and not none or all (‘a sausage contains some meat’); $||\geq \text{some}_1||$ means there are some or all (this quantifier is not used for meronymy). $||\text{several}||$ denotes the analogous quantifiers for a collection of objects, such as ‘a book contains several chapters’. The objects of a $||\text{several}||$-quantifier are always interchangeable (Chaffin & Herrmann call this ‘homogeneous’).

4 Quantificational tags for meronymy

The basic formal context for the investigation of semantic relations is a *denotative context* (Priss, in prep.3), which has denotata of words as formal objects and attributes of those denotata as formal attributes. The concepts can be denominated by disambiguated words in which case they are called *denotative word concepts*.

Two words w_1 and w_2 are in meronymy relation $w_1 m^r w_2$ if their word concepts B_1 and B_2 are in relation M^r to each other ($B_1 M^r B_2$). The four kinds of meronymy relations described by Cruse, can be formalized as follows. M_0^r is the facultative-facultative kind. (A child can be a member of a tennis-club, but not all children are members of tennis-clubs, nor do all tennis-clubs have children as members.) $M^r_{(\geq 0;\geq 1)}$ is the canonical-facultative kind. (All door-handles are part of a door, but not all doors have to have handles.) $M^r_{(\geq 1;\geq 0)}$ is the facultative-canonical kind. (All apartments have doors, but not all doors belong to apartments.) And, $M^r_{(\geq 1;\geq 1)}$ is the canonical-canonical kind. (Each bird feather is part of a bird, and each bird has feathers.)

The question of transitivity of meronymy, which has been widely discussed (Winston et al.), can be investigated with Relational Concept Analysis: it can not be answered in general, but it can be proved that, if r is transitive, then $M^r_{(\geq 0;\geq 1)}$, $M^r_{(\geq 1;\geq 0)}$, and $M^r_{(\geq 1;\geq 1)}$ are also transitive (Priss, in prep.1). It follows that the intransitivity of many meronymy relations is caused by the relational component r and not by the quantificational tag.

In the rest of this paper only the quantificational tags shall be investigated. Cruse's four kinds lead to the tags 0, $(\geq 0; \geq 1)$, $(\geq 1; \geq 0)$ and $(\geq 1; \geq 1)$. Table 1 contains the attempt to classify the meronymy relation by using quantificational tags. The examples are not complete. Missing combinations do not suggest that those examples do not exist, but that the author has not found them yet.

relational component	tag	example
stuff/object	($some_1$; ≤ 1)	meat/sausage
	($some_1$; 1)	sausage meat/sausage
	($\leq$ $some_1$; ≤ 1)	meat/pizza
stuff/mass	($some_1$; $\leq some_1$)	salt/seawater
	($some_1$; $some_1$)	sea salt/seawater
	($\leq$ $some_1$; $\leq some_1$)	sage/tea
	($\leq$ $some_1$; $some_1$)	sausage meat/food
element/mass	(several; $\leq some_1$)	body cells/skin
	(several; $some_1$)	body cells/body tissue
element/mass; portion/mass	($\leq$ several; $some_1$)	skin cells/body tissue; slice/bread
member/set	(several; ≤ 1)	tree/forest
	(several; ≥ 0)	human/citizenship
member/set; section/object	(several; 1)	human/sex; chapter/book
unit/measure; memb./set; obj./obj.	(n; 1)	sec./hour; card/deck; finger/hand
object/object	(≥ 0; 1)	refrigerator/kitchen
	(1; ≥ 0)	melody/song
obj./obj; individual/individual	(1; 1)	punch line/joke; Princeton/NJ

Table 1: A classification of meronymy

The first rows of the table show that the quantificational tags depend on the level of abstraction of the objects (compare, for example, meat/sausage and sausage meat/sausage). Furthermore, although meronymy relations with different relational components can share the same tags, each class of relational components tends to prefer a special tag. Therefore the tags can be a basis of a classification. The resulting classes differ from the four meronymy models of Iris et al., which distinguish membership, segmented whole, subset, and functional components. Here membership and segmented whole are in some cases closer together (compare human/citizenship and chapter/book). For the object/object relations, which correspond to the functional components, this classification seems to be the most unsatisfactory. For example, Chaffin & Herrmann's component/integral object, topological part/object, time/time, and place/area are all subsumed under object/object. Hopefully there will be a combination of research on tags and relational components in the future.

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