

# Associative and Formal Concepts

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**Abstract.** In several fields there is a divide between formal and associative models of concepts and reasoning. For example, in AI associative models such as neural networks and evolutionary computation are distinguished from symbolic, logic-based approaches. In psychology, fuzzy or category-based approaches compete with the “classical” theory of classification. In information science, systems based on dynamic, emergent structures can be distinguished from formal, manually designed structures. This paper argues that both modes of representation, formal and associative ones, need to be considered simultaneously for knowledge representation systems. This paper investigates the relationship between formal and associative structures and provides suggestions for bridging the gap between the two modes of representation.

## 1 Introduction

In many disciplines there is a dichotomy between formal and associative structures [19]. In psychology, Sloman [24], Pinker [18] and others have argued that there is evidence to support at least two co-existing systems of reasoning: an associative one and a rule-based (or formal) one. In AI, there is a divide between biologically inspired (i.e. associative) and logical-symbolical (i.e. formal) approaches. In cognitive science and linguistics, a “classical” formal model of concepts (which is sometimes called “Aristotelian” but that is misleading because Aristotle proposed different models) competes with a fuzzy, prototype-based model that can be traced back to Wittgenstein [28] and Rosch [22]. In information science, traditional, formal approaches led to the construction of information access systems such as library catalogs and web directories, for example, Yahoo!. Associative approaches in that field led, for example, to the search engine Google, which is not based on neural networks or fuzzy logic but is associative because of its dynamic reliance on the networking character of the WWW. In biology, the Whewell versus Mill debate focused on whether biological classes are prototype-based or can be defined using necessary and sufficient features (cf. [25], Chapter 6). The distinction between “formal” and “associative” is also similar to Wille’s [26] distinction between “mathematical” and “logical”. One might consider the traditional distinction between “light as waves” and “light as particles” in physics as an example of the same divide. In physics both are nowadays combined into one model in quantum mechanics. But in some other disciplines, researchers still debate whether ultimately both models can or should be combined and how.

In AI, artificial neural networks and evolutionary programming simulate associative cognitive abilities, such as stimulus generalization based on similarity and analogical reasoning, and evolutionary development in complex dynamic systems. Deacon

[7] provides a good overview of these models with respect to the neurophysiology of the human brain and the co-evolution of language and the human brain. A number of small-scale simulations have been developed that demonstrate the applicability of artificial neural networks to associative cognitive tasks, for example, Shastri's [23] model of rapid memory formation or Regier et al.'s [21] model of word/meaning associations. While these models successfully simulate isolated cognitive functions, it is not clear how these systems could scale up to automatically generate higher level cognitive abilities, such as logical reasoning and abstraction.

At the other extreme in AI are logic programming, rule-based expert systems and formal ontologies, such as Lenat's CYC [6]. These systems can easily represent abstract formal processes but even after 20 years of manual labor, CYC is still far from simulating the knowledge and intelligence of just a small child. To achieve artificial intelligence that is closer to human intelligence, perceptive interfaces may need to be connected both to low-level, associative neural networks but also to high-level, formal knowledge repositories, such as CYC.

It should be noted that the distinction between associative and formal structures made in this paper is essentially identical to Deacon's [7] distinction between indicative and symbolic representations or animal calls and human language, respectively. Thus Deacon's observations about the systematic character of language and the co-evolution of brain and language also apply to formal structures. But his book has one shortcoming in that he uses only one dimension (the sign dimension) with three classes in his structural analysis of indicative versus symbolic. This paper uses three dimensions (sign, object, internal representation) which result in ten combined classes and provide a larger set of differentiating criteria. For example, if only one dimension is used, it is difficult to explain why some animal behavior is more indicative, such as running away in fear, whereas other behavior is more symbolic or habitual, such as the nuances in dog barking that show habitually evolved meaning differences. Using three dimensions, it can be stated that running away and barking are different with respect to the sign dimension but equal with respect to the other two dimensions (see below). Similarly, barking and human language are equal with respect to one dimension and different with respect to the other two dimensions.

Section 2 of this paper describes the differences between associative and formal concepts. Section 3 develops a ten-fold classification of concepts that serves as a model for describing some of the features of and processes related to associative and formal concepts. Section 4 establishes some of the steps involved in the combination of formal and associative concepts which yield feedback loops that can explain the exponential difference in the conceptual abilities between the cognition of humans and other animals. The last section of this paper provides a brief sketch of possible applications of formal and associative concepts in ontologies and lexical databases.

## **2 The differences between associative and formal concepts**

Associative reasoning is grounded in a close connection of the individual's interaction with an external world. Clark [4] explains that humans do not have a complete model of the external world in their minds. Instead, our world model is continuously updated and

completed by perceptual input. Because of their connection to an external world, associative concepts and contexts contain a significant depth of detail. But they are usually restricted to a very narrow segment of time, space and culture. Both associative concepts and contexts are what Lenat [13] calls “rich objects”. That means they are infinite and rich in detail and cannot be completely described. For example, an utterance such as “birds fly” can invoke detailed visual representations of prototypical birds engaged in a prototypical activity in a speaker’s mind. But the utterance is usually not meant to be interpreted in a formal manner implying that all birds in some abstract, global context have the ability to fly.

Formal concepts, which are defined with respect to formal contexts, on the other hand, are somewhat abstracted from an external world. Wille [26] states based on Peirce that “mathematical thinking abstracts logical thinking with its basic forms of thought for hypothetically developing a cosmos of forms for potential realities”. Formal thinking thus facilitates contemplating hypothetical situations. Lakoff & Johnson’s [15] claim that all human philosophy has been shaped by our bodily experiences suggests that even formal thinking is on some level motivated by an external world. But Deacon [7] (p.87) explains that even though formal structures as a whole built on associative structures, formal structures form a system based on their internal connections which are stronger than their connections to associative structures. Because of their abstract nature, formal concepts and contexts tend to be more shallow or less detailed in their internal structures than associative concepts. But formal contexts also tend to be more globally defined than associative contexts because they depend less or not at all on temporal, spatial and cultural constraints. For example, while there is a historical context in which mathematical objects were first invented, their mathematical properties do not usually depend on that external, historical context.

In addition to hypothetical situations, formal concepts can represent objects that are recursive or on a meta-level. While it is not possible to imagine such objects purely in an associative manner, formal concepts themselves give rise to associations. For example, initially unicorns are defined formally because they do not exist. But people associate properties and images with unicorns. From a psychological viewpoint, unicorns can be represented in the human mind in the same manner as horses. This can even be true for mathematical objects. Devlin [8] states that although the objects of mathematics are formal, mathematicians tend to think about these formal objects in an associative manner. Formal and associative modes of thinking are thus highly intertwined in human cognition. In fact the connection between associative and formal structures may be one of the driving forces for human cognition. This connection is multi-functional and complex in design; has evolved over time; and may contain multi-level connections and feedback loops between the associative and formal processes.

## **2.1 Formal reasoning and cognitive activity**

The cognitive activities involved in associative and formal reasoning are different. Formal concepts and contexts are evaluated with respect to logical correctness, consistency and completion. Formal arguments do not require grounding but instead rely on logical inference. According to Ganter & Wille [11] and based on Kant, the three main cognitive activities of formal thinking are the definition and formalization of formal

concepts (“concepts”), the establishment of consistent relationships between concepts (“judgments”), and the investigation of entailments that arise from such relationships (“conclusions”). These encompass much of the current research in knowledge representation, such as modeling of ontologies and databases (concepts), information retrieval and database querying (judgments) and logic programming and automated reasoning (conclusions).

Formal reasoning can be formalized in its entirety. Wille [27] explains that all formal reasoning, consisting of concepts, judgments and conclusions can be fully described within a formal framework based on formal concept analysis. Formal reasoning is thus contained within the formal representations. But, of course, formal reasoning is just one mode of human reasoning, which normally uses formal and associative aspects simultaneously.

## 2.2 Associative reasoning and cognitive activity

Associative concepts and contexts are evaluated with respect to their grounding. A speaker who uses an associative argument first attempts to establish an associative context within the listener’s mind in which the associative concepts are then embedded. The listener can understand the speaker’s argument if the new associative structures fit with structures the listener has already established. The listener is convinced that the speaker’s argument is sound, if the new associative structures resonate with the listener’s thinking. It is irrelevant whether the statements are “true” or not. An example for this difference between associative and formal arguments is the fact that it is possible to follow the logical structure of a mathematical proof and to be convinced of its correctness but to still not understand its meaning if the proof does not resonate with prior knowledge.

Associative reasoning in general employs other cognitive activities than formal reasoning. First, associative concepts need to be selected or identified, usually in the form of patterns or gestalts. While formal concepts are defined and formalized, associative concepts are selected from an infinite number of possibly interesting gestalts and patterns that arise from interaction with an external world. Artificial neural networks can be trained to perform this task of identifying associative concepts within limited domains. Associative concepts cannot be defined using formal concepts or linguistic expressions but different types of representations can be associated with associative concepts. The physical form of a representation of an associative concept matters, or, in other words connotations that are associated with a representation can influence the associative concept itself. Finding an appropriate representation for an associative concept is thus part of identifying such a concept. This is in contrast to formal concepts, which always must be represented in a symbolic language but only depend on the formal relations that are established by the representation not on the forms of individual representations. This is the reason why mathematics can easily be translated into different languages but poetry cannot easily be translated.

A second cognitive activity of associative reasoning relates to the transfer and expansion of conceptual structures. Associative reasoning does not employ logical inference but instead analogy and metaphors. Hofstadter [9] identifies analogy as “the core

of cognition”. Analogy is not a precise method but instead involves the details and richness of objects of an external world. Peirce [17] (p. 227) states that “deduction consists in constructing an icon or diagram [of] the relations [...] whose parts shall present a complete analogy with those parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts”. His claim is that deduction in human cognition is not an entirely formal method but contains what is called associative reasoning in this paper. Associative reasoning in the form of analogy involves the establishment of relationships between objects and their parts in an external world. Peirce’s view is thus not so different from the modern view of analogy as structural alignment [12].

There may be fewer formal methods and fields of research devoted to associative reasoning than to formal reasoning. Related fields are among others artificial neural networks and evolutionary computation in AI; data mining with its interest in automatic identification and selection of potential concepts and relationships; and the ancient art of rhetoric, which includes the study of how to exploit associative arguments.

Human reasoning usually involves both associative and formal methods in combination because without associative concepts, reasoning would have no ground and no relationship to an external world. Without formal concepts, reasoning would be limited to objects within the actual physical environment and their evident features and relations. No broader consequences, abstract structures or possibilities could be considered. Devlin’s [8] main conclusion about what differentiates people with and without a mathematical ability is that mathematicians are capable of thinking about mathematical objects in the same gossip-like (and thus associative) manner as other people think about soap operas. Bauer [2] explains that science itself does not follow the formal model of the “scientific method” but instead also heavily depends on certain other social and instrumental (and thus associative) factors. It is thus important that formal models about reasoning include both the formal and the associative aspects of reasoning.

### **3 A classification of concepts**

This section presents a classification of concepts (see figure 1) which serves to highlight some of the differences between formal and associative concepts. The classes provide a model for a stepwise evolution from associative to formal concepts. The classification has three dimensions, each with three classes. In figure 1 the first dimension refers to the types of objects and is represented vertically. The second dimension refers to the types of signs and is represented horizontally. The last dimension distinguishes types of internal representations and is represented by three areas: classes 1-3 at the top, classes 4, 5, 7, 8 in the middle, and classes 6, 9, 10 on the right.

Normally three dimensions with three classes each would yield a direct product of 27 classes. But there are dependencies between the different dimensions which reduce the number of classes to 10. This classification is structurally equivalent to Peirce’s ten-fold classification of signs but the content of the classes is different. The structural equivalence may be due to the fact that in both cases the three dimensions contain three classes which are increasing in complexity i.e. what Peirce calls Firstness, Secondness and Thirdness.

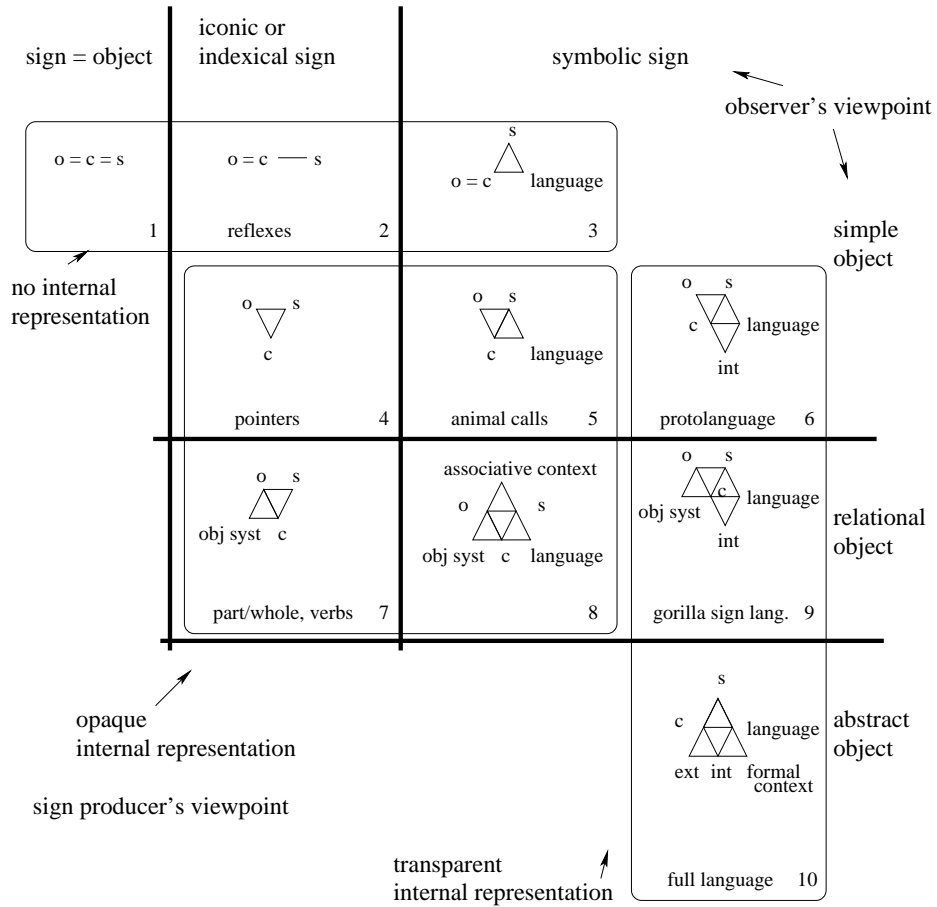


Fig. 1. A classification of concepts

### 3.1 The object dimension

The three classes of the object dimension are simple objects, relational objects and abstract objects. This dimension refers to an external viewpoint of an observer who observes these objects within an associative context. The classes are thus not intrinsic features of objects but based on the judgments by an observer.

Simple objects are gestalt-like structures or patterns in an external world. Examples are “stone”, “hot”, “yellow”, and “three”. Simple objects are similar to what Lakoff [14] calls “basic-level structures”. He states that basic-level structures arise “as a result of our capacities for gestalt perception, mental imagery, and motor movement” (p. 302). Even though “hot” and “yellow” are fuzzy when used in language and vary among different speakers and situations, they correspond to simple physiological gestalts: “hot” corresponds to an unpleasant heat sensation and “yellow” to one of three color receptors in the human eye. Devlin [8] explains that the numbers one, two and three are perceived in an immediate manner by many animals and by humans and do not require a counting ability. In figure 1, simple objects are denoted by  $o$ .

Following Lakoff & Johnson’s [15] argument about the embodiment of cognition, it is conceivable that gestalt perception is a deterministic property of an external world constrained only by the physical, bodily properties of perception. That means that beings with similar bodies and perceptive mechanisms are capable of perceiving similar simple objects. The notion of “objects in an external world” is to be understood in this manner. Principles of gestalt perception have been established by psychologists and can be simulated using artificial neural networks. The more challenging aspect of gestalt perception is not to form gestalts but to select the ones which are interesting for or relevant to an individual in a situation. (This is one of the main challenges for data mining applications.)

Relational objects are objects that consist of relations among objects. Examples of relational objects are part-whole relations, many prepositions and verbs. For example, “over” is a relational object that consists of a relation between two simple objects. These relations are usually identified with respect to an “external world” or “object system” (denoted by “obj syst” in figure 1). To recognize such relations, some kind of internal representation (or conceptualization) is required, thus relational objects are established in a triad of object system, object and conceptualization (denoted as  $c$ ) in figure 1.

In contrast to simple and relational objects, abstract objects are always culturally determined. They are defined as objects that are under no circumstances directly emergent from an external world but have components that are culturally created and require interpretation. Examples are the abstract notions of “mathematics” and “democracy”. Typically it is possible to represent simple and relational objects in an iconic or indexical manner. But abstract objects must be represented symbolically using the symbols of a language. In figure 1, abstract objects are represented by  $ext$  which stands for “extension of a formal concept”. Animals, which have similar body size and perceptive abilities as humans, are most likely capable of perceiving the same simple and relational objects as humans. But they may not choose to be interested in the same objects and most likely they cannot contemplate abstract objects.

### 3.2 The sign dimension

The three classes of the sign dimension are “sign = object”, iconic or indexical and symbolic. This dimension also refers to an external viewpoint of an observer who observes associations between signs and objects within an associative context. There is no intrinsic difference between signs and objects because signs usually have a physical representation and are thus physical objects. But in an associative relation between an object and a sign, it can be identified which one is the sign and which one is the object based on the focus of attention. According to Regier et al. [21], objects are whatever is the focus of attention and signs are whatever is associated with an object but not the focus of attention.

The first class, “sign = object”, in this dimension refers to associative relations in which the sign and the object are essentially identical. Examples are contexts in which an observer views an object without any interpretation or intention and no communication is involved. Only simple objects can be viewed in such a manner.

Iconic and indexical signs are grouped together in the next class because both involve a physical or causal relationship between sign and object that is grounded in an associative context. For example, iconic similarity is based on observing shared features between object and sign, which is a physical relationship. Pointers establish causal relationships between signs and objects. Due to the physical or causal relationship, an observer needs no further information (such as linguistic knowledge) to identify the relation between object and sign but, of course, causality is observer-dependent. Signs in this class can represent both simple and relational objects, but not abstract objects. An example of the use of indexical signs among animals is how wolves use a complex communication system of pointing with their gaze and the direction of their muzzles during hunting activities [5]. The wolves communicate relational objects such as “come”, “sit down” and “go there” in that manner.

Symbolic signs are signs that are part of an actual language, such as a human language. Therefore in figure 1, symbolic signs are represented by triads of signs, conceptualizations and language. A characteristic of symbolic signs is that they are habitual and cannot be understood without knowledge of the language. Complex systems of animal calls, such as the different barks a dog can produce [5], also fall into this class because they cannot be understood without knowledge of the code. Dog barking is a complex habitual sign system - although it is mostly innate and not conventionally defined such as human languages. An obvious difference between dog barking and human language is that dogs cannot communicate abstract objects but humans can. Thus some symbolic sign systems can represent all types of objects (simple, relational and abstract ones), others can only represent simple ones or simple and relational ones.

### 3.3 The dimension of internal representations

The third dimension pertains to the internal representations or conceptualizations (denoted by *c* in figure 1) that mediate between the perception or contemplation of objects and the production of signs. “Internal representation” in this paper refers to the existence of an internal brain-like or higher-order neural representation within the sign



producer. This dimension does thus not pertain to the viewpoint of an observer but instead to the viewpoint of a sign producer. But observers often have some limited means of determining the existence of internal representations of other sign producers based on certain clues (see below).

In the first class, there is no internal representation. This is denoted by  $o = c$  in figure 1. Examples of the lack of internal representations are a sunflower turning to the sun, communication among bacteria or hormonal communication. These processes are entirely deterministic or of the stimulus/response type without an opportunity for choices. Learning can only occur at the system level through evolution but individuals cannot learn during their life-time and cannot change their behavior. In this class an external object as input to an agent is directly (although possibly with temporal delay) followed by the output of a predetermined sign. This sign can be in different classes, for example, it can involve a direct causal relationship such as fear/sweat or reflexes, or can be symbolic such as in the case of hormones.

In the second class, the internal representations are opaque from the sign producer's viewpoint. In figure 1 this is denoted by a triad consisting of an object, a sign and an internal representation ( $c$ ). From an observer's viewpoint there is evidence for the existence of internal representations provided by the fact that the sign producer appears to have choices. The sign producer does not appear to react according to simple stimulus/response mechanisms or deterministic input/output processes. Instead the sign producer's behavior is influenced by subtle contextual changes in a complex manner. But there is no evidence that the sign producers at this level can reason about their internal representations. Cognitive abilities that can be achieved in this class are stimulus generalizations (i.e., objects can be grouped into categories) but the category boundaries are fuzzy and based on prototypes. For example, current artificial neural networks can learn to categorize simple and relational objects but the networks cannot also output the reasons why they categorize in a certain manner. The symbolic signs produced by opaque internal representations are limited to one-word statements, such as produced by 1-2 year old children and most animal calls.

The last class contains transparent internal representations which means that the sign producer appears to have some insight into her internal representations. From an observer's viewpoint, the evidence for this is the fact that the sign producer can build simple syntactic combinations of signs in the form of object/attribute (object HAS attribute) or object/class (object ISA class) associations. Instead of simply associating objects and signs, the sign producer can thus express some reasons why objects and signs are associated, which eventually leads to the ability to produce formal definitions in class 10. Obviously, transparent internal representations depend on the existence of some language. In figure 1, transparent internal representations are denoted by an association of  $c$  and  $int$  which represents the intension of a formal concept. The symbol system at this stage is called "protolanguage" [8]. That means it contains simple sentences of subject-verb structure but without nesting. Apart from humans maybe only apes can reach this ability. For example, gorillas can form simple 4- to 6-word sentences in sign language and thus explicitly express relationships between objects and attributes [16]. The notion of "transparency" is not meant to imply that at this level all concepts are fully transparent to sign producers or that they can be consistently defined.

Concepts at this level may only exist in the sense of prototype theory [22]. But “transparency” means that humans are capable of contemplating at this level about what their concepts are made of. Transparent internal representations require the use of symbolic representations because symbols are required to express the intensions.

Only abstract objects (class 10) require transparent internal representations and full language (as opposed to protolanguage). It is unlikely that other animals apart from humans are capable of contemplating abstract objects or using full language. For example, it is unlikely that apes can be trained to fully understand what “democracy” means. The difference between class 9 and 10 is that in 10 both the extensions and the intensions of concepts are expressed as signs without direct reference to objects or object systems. These concepts correspond to formal concepts in formal concept analysis [10]. Deacon [7] (p. 83) states that “Words also represent other words. In fact, they are incorporated into quite specific individual relationships to all other words of a language.” and “This referential relationship between the words [...] forms a system of higher-order relationships.” At this level formal contexts emerge which form systems that are somewhat independent of associative contexts. Formal contexts consist of formal concepts that are defined in terms of extensions and intension, which are all represented using symbolic signs of a language.

Devlin [8] (p. 219) explains that full language facilitates off-line thinking, i.e. thinking about objects that are not necessarily part of the immediate physical environment. This supports the idea that the extension of concepts in class 10 does not contain objects but instead signs. Signs in an extension facilitate the expression of meta-level statements and recursion. In all other classes (1-9) it is not possible, for example, to express “the word *word*”. Class 10 also facilitates nesting, such as “I believe that ...” or “John says that ...”, and hypothetical statements and other complex structures because the signs in an extension can provide constraints for the concepts. Class 10 thus supports sign systems with full syntax.

Technically, recursive language development must have some starting points. That means that some concepts of a language must be grounded and belong into the classes 5, 6, 8 or 9. But this grounding occurs at a systematic level not at an individual level. That may be the reason why even though linguists have undertaken many attempts to identify “primitive” concepts, it has not been possible so far to generate an ultimate list of primitives. Another problem is that concepts can change their nature and migrate from class 10 to other classes. For example, unicorns are originally class 10 because they do not exist and have been invented by humans. But the external world to which an object *o* belongs is not limited to the physical world. Humans can invent imaginary objects and provide them with a virtual existence (shape, color, characteristics) that is promoted via images and stories. Therefore unicorns can even belong to class 5, if they are graphically represented and grounded into an appropriate associative context. That means formal contexts can be used to generate and modify object systems.

#### **4 The power of combining associative and formal concepts**

The purpose for developing this ten-fold classification of concepts is to provide a model for associative and formal concepts. Classes 1-3 do not contain concepts. The concepts

in classes 4-9 are associative; the concepts in class 10 are formal. A major aspect of human cognition is not alone the existence of formal concepts, which by itself is a significant difference between the cognitive abilities of humans and other animals, but the seemingly exponential power of generating new concepts that arises from the combination of associative and formal structures. Clark [4] uses the notion of a “mangrove effect” to denote a positive feedback loop in cognitive processes. In the same manner as mangroves create land which encourages further mangroves to grow which create more land, cognitive processes shift between representations, such as external and internal ones or formal and associative ones, to achieve exponential growth of capabilities. For example, mathematicians often draw diagrams on a piece of paper while they are solving formal problems.

It is reasonable to assume that mangrove effects apply to associative and formal structures. For example, formalizing phenomena that have been associatively observed often leads to completely new strains of thought, discovery of new patterns, and new sets of associative concepts which then again can be formalized and so on. A combination of formal and associative structures is essential to exploit the grounding of associative concepts, the logical, recursive power of formal concepts and the feedback loops that combine both. But it is still an open question how to build systems that implement such a combination. It is not yet well understood how artificial neural networks could reach a level of formal concepts, or how formal systems such as CYC could interact with a perceptive input device and neural networks.

Using the classification developed in the last section it is possible to identify some of the potential steps that are involved in an evolution from associative to formal concepts and also to identify the points where feedback loops point back from formal structures to associative ones. The main path from associative structures to formal structures starts with gestalt perception and pattern recognition (classes 4, 7). Driving forces for these classes are interaction with an external world as represented by an object system and communication with others. Both influence focus and attention. Concepts at this level often combine associations of co-occurring perceptive inputs, such as sound and visual perception. The association of signs and simple and relational objects (classes 5 and 8) is most likely also based on repeated co-occurrence. Regier et al. [21] describe an artificial neural network that provides a model for the emergence of words in classes 5 and 8.

The next step from classes 5 and 8 to classes 6 and 9 involves what Lakoff [14] calls the construction of *image schemata* and what is represented as intensions in figure 1. Image schemata provide schematic representations of the meanings of concepts. This is a first point where a feedback loop can take place. Image schemata and metaphors can be used to transfer known structures into new domains and establish new gestalts and pattern. Lakoff states that “in domains where there is no clearly discernible preconceptual structure to our experience, we import such structure via metaphor”. Experiments with gorillas which are taught sign language [16] seem to indicate that these gorillas are capable of forming simple metaphors spontaneously, such as using the signs for “dirty” or “toilet” to denote unpleasant situations or people.

The step from associative concepts (classes 6 and 9) to formal concepts (class 10) involves abstraction and quantification. Abstraction means that formal concepts are pri-

marily related to extensions which are signs themselves but not directly to objects. On an associative level, quantifiers are not specified. Even an associative ISA relation does not imply quantification. For example, the statements “dogs are pets” and “pets are dogs” can both be uttered in an associative context without any consideration as to whether these statements are true for *all* dogs in the context. In a formal context, however, relationships can be quantified. It can be stated that “*some* dogs are pets” and that “*all* dogs are mammals”. Precise quantification facilitates precise logical inferences as opposed to fuzzy associations.

As indicated before, class 10 facilitates a second feedback loop because the objects of formal concepts themselves can become virtual objects in imaginary worlds (or object systems) and give rise to associative concepts in classes 4 - 9. Because there are infinitely many possibilities to combine signs to represent hypothetical objects, this process can generate even more associative concepts than the application of image schemata (class 6 and 9) to new domains, which is still constrained by properties of the old and new domains. Besides the interactions mentioned here there are most likely numerous other interactions, such as lexical relations, which further add to the complexity and creativity of human cognition.

## 5 Outlook: a possible application to lexical databases

Computational linguists usually represent the whole complexity of semantic relations within linguistic structures. For example, Pustejovsky [20] proposes solutions that resolve the presumed polysemy of the word “finish” in sentences such as “Mary finished the cigarette” and “Mary finished her beer”. Pustejovsky claims that “the exact meaning of the verb *finish* varies depending on the object it selects”. The differences between the two sentences are thus considered a linguistic phenomenon.

The distinction between associative and formal concepts as presented in this paper suggests a different approach. The difference between finishing a cigarette and finishing a beer results from the differences between the associative contexts of cigarettes and beers. Every language user who knows what cigarettes are and what beer is and what an image schema for “finish” is will immediately understand what is meant by “finishing a cigarette” and “finishing a beer”. A language user will know that finishing a cigarette involves extinguishing it whereas finishing a beer involves emptying a glass. This is not lexical knowledge but instead knowledge that is based on the grounding of “beer”, “cigarettes” and “finish” into an external world. A theory of the ground of associative concepts is provided by sciences, such as physics, which explain the differences between gas or smoke and fluids. Any attempt to formalize such knowledge in a lexical format (such as Pustejovsky’s Generative Lexicon or CYC) would have to include all scientific and all common sense knowledge. On the other hand, if knowledge about associative concepts was separated from linguistic structures and formal knowledge, lexical databases (or ontologies) would be smaller and easier to manage.

Linguists have identified “regular polysemy” [1] as a phenomenon where several words have polysemous senses that are distinguished in a semantically similar manner, such as plant/food (eg. “to grow wheat” versus “to eat wheat”). Regular polysemy has been extensively investigated with respect to the lexical database WordNet (cf., [3]).

Regular polysemy can be automatically detected in WordNet because of the semantic relationships that are explicitly represented in WordNet. But regular polysemy only applies to a small part of the vocabulary.

We conducted a small scale analysis of WordNet's most polysemous nouns and verbs. The most polysemous noun is "head" with 30 senses; the most polysemous verb is "break" with 63 senses. We found that the senses fall into two categories: a) variations of some basic image schema that underlies a word and b) specialized uses of a word within fixed contexts or phrases. Phrases are not usually rule-based and thus must be listed in a lexical databases. A problem with respect to phrases in WordNet is that semantic relations are usually attached to the word instead of being attached to the phrase. For example, "break" is considered a hyponym of "dance" because of the sense "break dance, break". We found that about 25 % to 50 % of all senses in WordNet are phrases.

With respect to a), it is usually possible to reduce the remaining 50 - 75 % senses of a word in WordNet to 2 - 8 primary senses that relate to a basic image schema. The relations among the senses related to each image schema are fairly deterministic with respect to underlying associative concepts but not as easy to exploit as regular polysemy, which can be detected via explicit semantic relations in WordNet. Some examples of image schemata are nouns that have a container image schema. These usually have at least three senses: one for the container, one for the content and one for both. Nouns that are derived from verbs usually have senses related to the case frames of the verb: the actor, the object, the outcome, etc. Many of the more polysemous nouns have one sense which is used as a unit of measurement such as "300 head of sheep", "3 points on a scale", "3 feet tall", and so on. A better understanding of associative concepts would thus facilitate an improvement in the implementation of lexical databases.

## 6 Conclusion

There has been a recent increase in interest in associative and formal concepts, which have been identified as a dichotomy in several disciplines. This paper provides a model for distinguishing formal properties of associative and formal concepts. An understanding of the ways in which formal and associative concepts combine in human cognition would have major impact on the development of artificial intelligent devices. This paper provides hints as to where feedback loops between associative and formal concepts could be initiated and a sketch of where such structures would be useful in the design of lexical databases.

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