Relations and matrices

SET07106 Mathematics for Software Engineering

School of Computing
Edinburgh Napier University
Module Leader: Uta Priss

2010

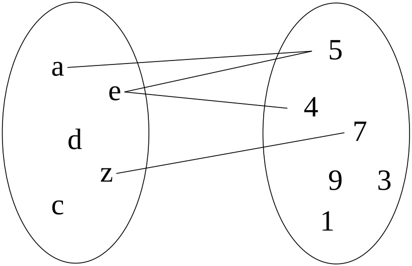
Outline

Binary relations

Properties

N-ary relations

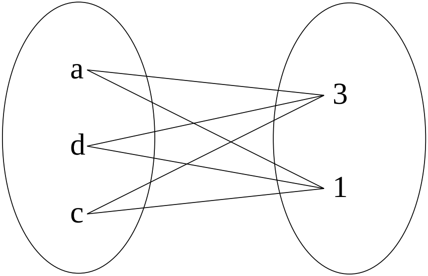
Binary relations

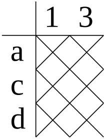


	1	3	4	5	7	9
a				X		
c						
d						
e			X	X		
z					X	

{ (a,5), (e,5), (e,4), (z,7) }

Cartesian product

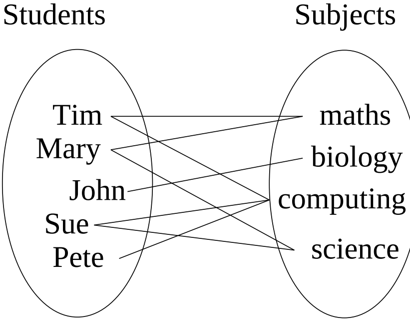




{ (a,1), (a,3), (c,1), (c,3), (d,1), (d,3) }

A complete bipartite graph. $|A| \times |B|$ elements.

Another example



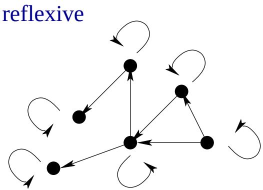
Students	Subjects
Tim	maths
Tim	computing
Mary	maths
Mary	computing
John	biology
Sue	computing
Sue	science
Pete	computing

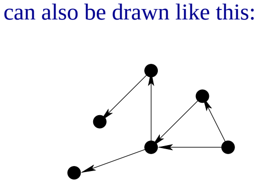
Properties of binary relations

- reflexive: for all elements: (a, a)
- symmetric: $(a, b) \iff (b, a)$
- antisymmetric: $a \neq b : (a, b) \implies \text{not}(b, a)$
- transitive: $(a, b), (b, c) \implies (a, c)$

Reflexivity

For all elements: (a, a)

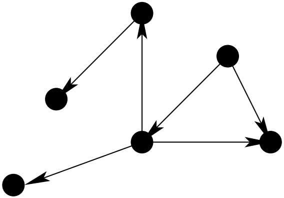




Directed graphs (digraphs)

$$a \neq b : (a, b) \implies \text{not}(b, a)$$

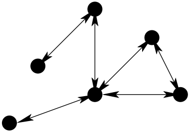
antisymmetric:

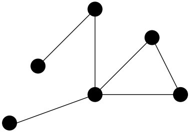


Undirected graphs

$(a, b) \Longleftrightarrow (b, a)$

symmetric can be drawn as an undirected graph:

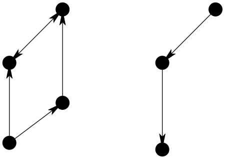




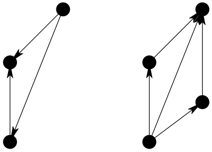
Transitivity

$$(a, b), (b, c) \implies (a, c)$$

intransitive:



transitive:



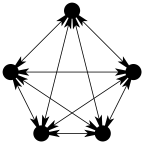
Equivalence relation (or partition)

An equivalence relation is a relation that is

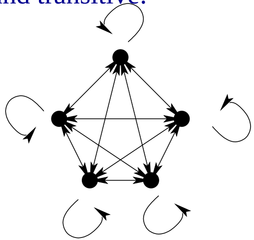
symmetric, reflexive and transitive.

Equivalence relation: complete graph

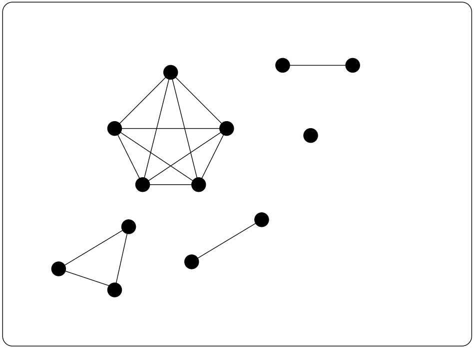
symmetric
and transitive:



reflexive, symmetric
and transitive:



Equivalence relation: set of complete graphs



Exercises

Determine the properties of these relations and draw a graph for each relation.

- $R_1 = \{(1,1),(1,2),(2,2),(2,1),(3,3),(4,4)\}$
- $R_2 = \{(1,1),(1,3),(2,2),(2,1),(3,1),(1,2)\}$
- $R_3 = \{(a,b),(b,c),(c,d),(d,e),(e,a)\}$
- $R_4 = \{(a,b),(b,c),(a,c),(d,e),(f,g)\}$
- $R_5 = \{\}$
- $R_6 = \{(a,a)\}$

Ternary and n-ary relations

For example a relational database:

name	street	city	phone	billAmount
Mary Smith	Colinton Road	Edinburgh	123 4567	15.00
Paul Jones	London Road	Edinburgh	123 8765	17.00
Paul Jones	London Road	Edinburgh	123 3926	21.50
Tim Taylor	Colinton Road	Edinburgh	123 6385	15.00
Susan Miller	Baker Street	London	345 5932	7.00

Key attributes:

Ternary and n-ary relations

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Tim Taylor	Colinton Road	Edinburgh	123 6385	15.00
Susan Miller	Baker Street	London	345 5932	7.00

Key attributes: name and phone

Ternary and n-ary relations

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Paul Jones	London Road	Edinburgh	123 3926	21.50

The relations in relational databases are functions on the key attributes.

street('Mary Smith') = 'Colinton Road'

billAmount('Paul Jones', '123 8765') = 17.00

Binary functions: matrices

A special kind of a ternary relation is a binary function, which can be represented as a matrix.

Table:

	1	2	3
a	15	2	17
b	1	10	16
c	4	5	17

Matrix:

$$\begin{pmatrix} 15 & 2 & 17 \\ 1 & 10 & 16 \\ 4 & 5 & 17 \end{pmatrix}$$

Ternary relation:

{ (a,1,15), (a,2,2), (a,3,17), (b,1,1), (b,2,10), (b,3,16), (c,1,4), (c,2,5), (c,3,17) }

Binary function:

f(a,1)=15, f(a,2)=2, f(a,3)=17, f(b,1)=1, f(b,2)=10, f(b,3)=16, f(c,1)=4, f(c,2)=5, f(c,3)=17

Matrix operations

Addition:

$$\begin{pmatrix} 1 & 0 & 0 \\ 5 & 6 & 0 \\ 0 & 3 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 \\ 7 & 2 & 3 \\ 0 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 12 & 8 & 3 \\ 0 & 7 & 4 \end{pmatrix}$$

Multiplication:

$$\begin{pmatrix} 1 & 0 & 0 \\ 5 & 6 & 0 \\ 0 & 3 & 4 \end{pmatrix} \circ \begin{pmatrix} 0 & 1 & 0 \\ 7 & 2 & 3 \\ 0 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 42 & 17 & 18 \\ 21 & 22 & 9 \end{pmatrix}$$