Small World Networks

SET07106 Mathematics for Software Engineering

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Outline

Introduction

Watts & Strogatz

Scale-free networks

Clustering Coefficient

This lecture is partly based on the book
"Small World" by Mark Buchanan.

Stanley Milgram's Small-World Experiment

Sending packages to a stockbroker in Boston by sending them to random people in Nebraska and asking them to forward to someone who might know the stockbroker.

$\Rightarrow$ Six degrees of separation

Erdös number

Mathematician Paul Erdös published more papers than any other mathematician in history and had hundreds of co-authors.

Co-authors have Erdös number 1.

Co-authors of co-authors have Erdös number 2.

90 percent of the world's active mathematicians have an Erdös number smaller than 8.

It has been said: Someone who does not have an Erdös number is not a mathematician.

But: not every network is a small world

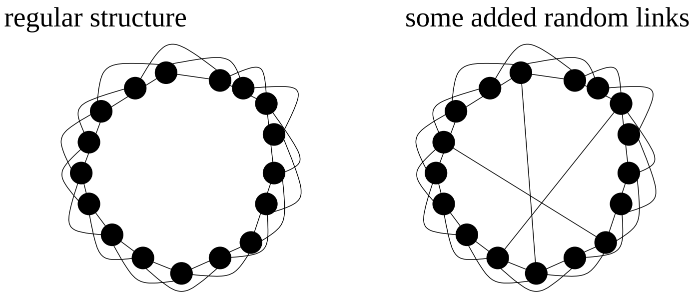
For example: Six degrees only relevant for living people.

Distance between people who lived hundreds of years apart is much larger.

Two types of networks

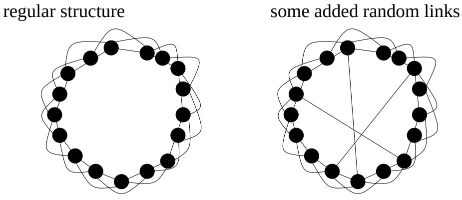
- Watts & Strogatz networks
- Scale-free networks

Watts & Strogatz's networks



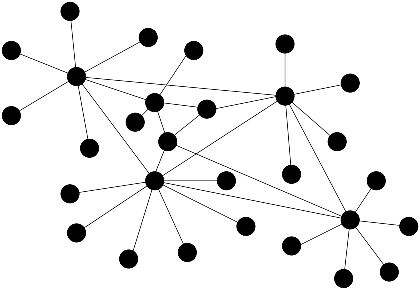
Examples: electric power grid, neuron connections in the brain

Watts & Strogatz's networks



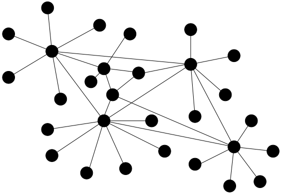
- high local clustering with a few distant links
- egalitarian
- no hubs

Scale-free networks



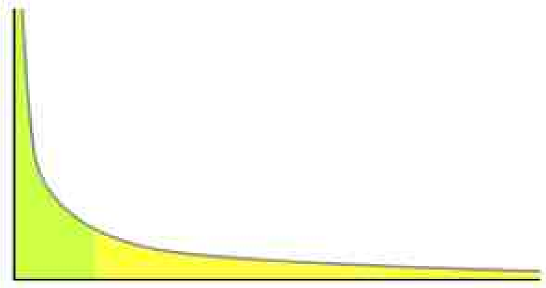
Examples:
WWW, citation networks, airline networks, biological networks

Scale-free networks



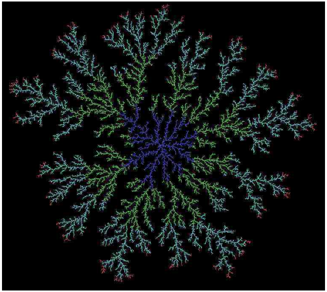
- hubs and resources
- power law distribution
- growths: preferential attachment ("the rich get richer")

Power law distribution



Examples: word frequencies, earthquakes, popularity of movies hubs and resources on the WWW

Diffusion limited aggregation



⇒ fractal growths, self-similarity,
preferential attachment ("the rich get richer")
Example: river networks, hubs on the WWW

Comparison of small world network types

Watts & Strogatz	scale-free
egalitarian	hubs and resources
no growth	preferential attachment
local clusters and distant links	power law
under attack: deteriorates	under random attack: stable
no particular targets	under targeted attack: weak

Both: small average node-to-node distance

Application: disease control

Diseases are spread by "hubs".
$\Rightarrow$ Outbreaks can be controlled by treating the hubs.

Examples:

- Treatment of rabies in foxes:
  vaccinate the ones that are travelling long distance.
- Influenza: vaccinate children.

Clustering Coefficient

$$\text{Clustering for node } n = \frac{|\text{ actual edges between neighbours of } n|}{|\text{ possible edges between neighbours of } n|}$$

Number of possible edges between k nodes: $\frac{k(k-1)}{2}$
(I.e. number of edges in a complete graph with k nodes)

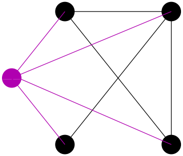
Clustering coefficient for node n with k neighbours

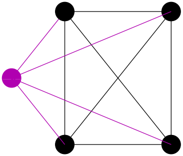
$$C(n) = \frac{|\text{actual edges}|}{\frac{k(k-1)}{2}} = \frac{2 \times |\text{actual edges}|}{k(k-1)}$$

Clustering Coefficient

neighbours

node



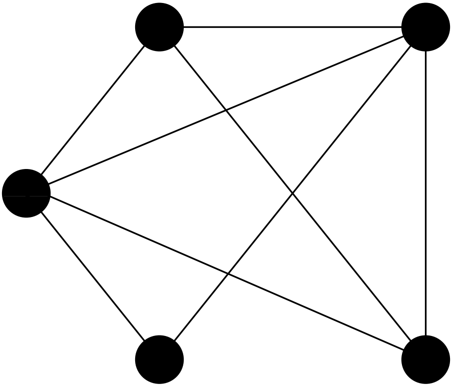


4 actual edges

$$\frac{4 \text{ x } 3}{2} = 6 \text{ possible edges}$$

$$\text{Clustering : } \frac{4}{6} = 0.66$$

Exercise: Calculate the average clustering coefficient



Exercise: Calculate the average clustering coefficient