

Functions

SET07106 Mathematics for Software Engineering

School of Computing
Edinburgh Napier University
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2010

Outline

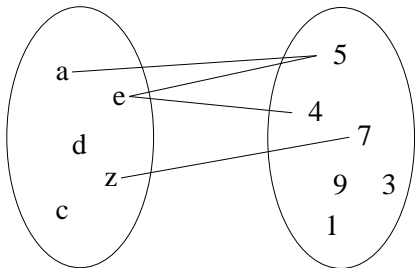
Functions

Real functions

Python functions

Programming with functions

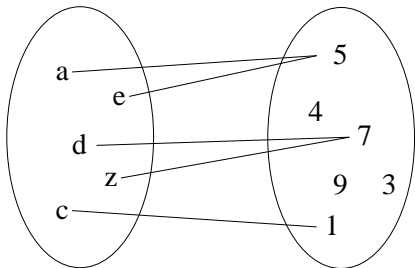
A relation between two sets: a binary relation



	1	3	4	5	7	9
a				X		
c						
d						
e			X	X		
z					X	

{ (a,5), (e,5), (e,4), (z,7) }

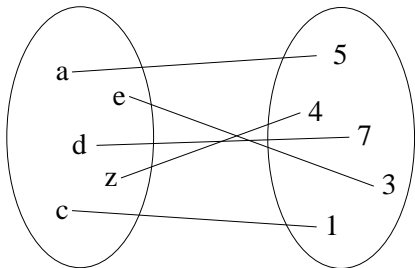
A function: mapping from one set into another set



	1	3	4	5	7	9
a				X		
c	X					
d					X	
e				X		
z						X

{ (a,5), (e,5), (d,7), (c,1), (z,7) }

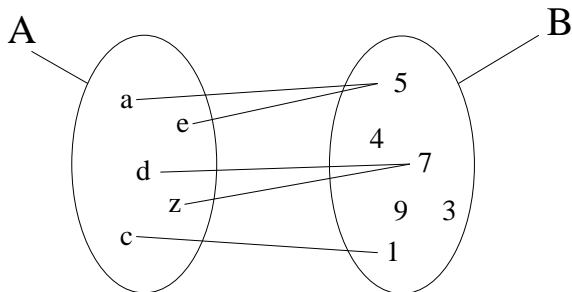
A one-to-one function: a bijection



	1	3	4	5	7
a				X	
c	X				
d					X
e		X			
z			X		

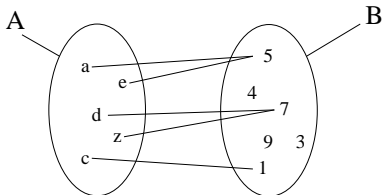
{ (a,5), (e,3), (d,7), (c,1), (z,4) }

A function: mapping from one set into another set



$$f : A \rightarrow B$$

Notations and terminology for functions



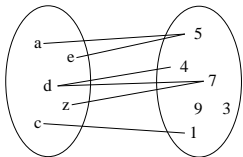
- ▶ $f : A \rightarrow B$
- ▶ $f(a) = 5; f(e) = 5; f(d) = 7 = f(z); f(c) = 1$
- ▶ A is called **domain**
- ▶ B is called **co-domain**

Definition

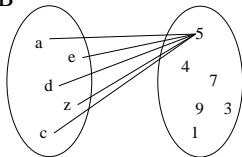
A function is a relation where **each** element in the domain is associated with **exactly one** element in the co-domain.

Which of these are functions, bijections, relations?

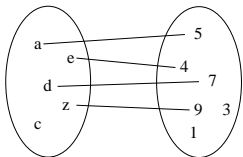
A



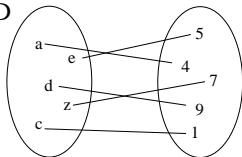
B



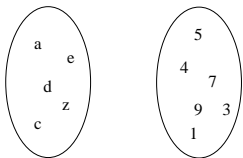
C



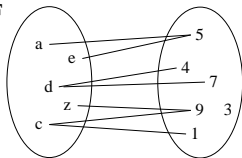
D



E

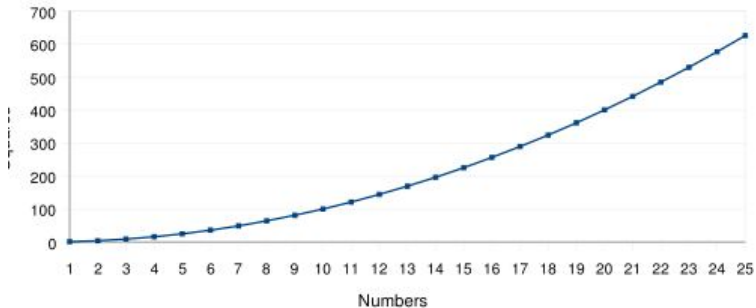


F

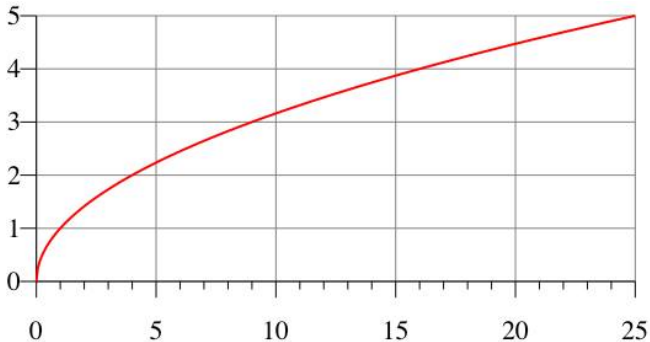


Examples: $f(x) = x^2$

Numbers and their squares

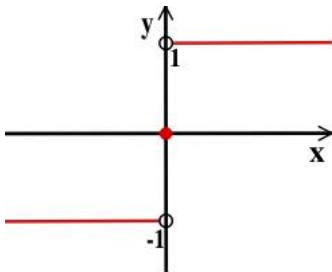


Examples: $f(x) = \sqrt{x}$

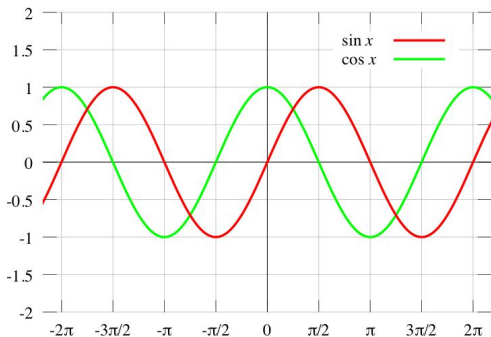


Examples: sign function

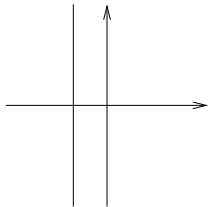
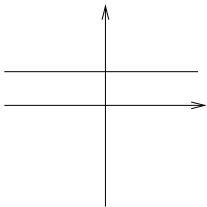
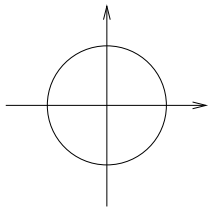
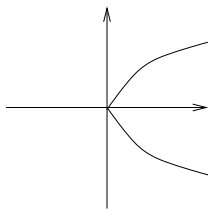
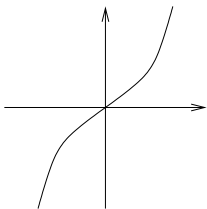
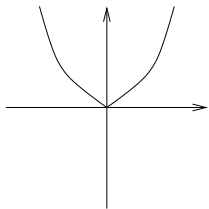
$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$



Examples: $f(x) = \sin(x)$, $f(x) = \cos(x)$



Which of these are functions, bijections, relations?



Python functions

- ★ What are the domains, codomains of these functions?
- ★ What are the results?

```
a = "abc"  
len(a)  
bool(a)  
max(a)  
min(a)
```

Methods are (often) just functions

★ In Python:

```
a = "abc"  
a.capitalize()
```

★ In mathematical notation:

$$f("abc") = "Abc"$$

Some Python methods for strings

- ★ What are the domains, codomains of these functions?
- ★ What are the results?

```
a = "abc"  
a.capitalize()  
a.endswith("c")  
a.find("b")  
a.isalpha()  
a.isdigit()
```

... and many more.

SymPy's mathematical functions

```
sqrt(4)
sign(-1)
factorial(7)
factorial(7) == 1 * 2 * 3 * 4 * 5 * 6 * 7
N(sin(1))
```

... and many more.

Defining a function in Python

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

```
def sign_function(x):  
    if x < 0:  
        return -1  
    elif x == 0:  
        return 0  
    else:  
        return 1
```

```
sign_function(8)
```

Defining a recursive function

Reminder: $\text{factorial}(7) == 1 * 2 * 3 * 4 * 5 * 6 * 7$

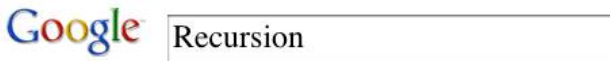
```
def fact(n):  
    if n == 0 or n == 1:  
        return 1  
    else:  
        return (n * fact(n-1))  
  
fact(7)
```

A dictionary definition:

Recursion

See Recursion

A Google joke:



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Did you mean: [*Recursion*](#)

[Recursion](#) - Wikipedia, the free encyclopedia

A visual form of **recursion** known as the Droop